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Characteristic matching between stator and rotor in standing-wave-type ultrasonic motors

Jiamei Jin · Chunsheng Zhao

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Abstract This work discusses kinematical and dynamic matching between stator and rotor in standing-wave-type ultrasonic motors. A remarkable feature of standing-wave-type ultrasonic motors is that the stator intermittently contacts the rotor. The dynamic responses of the rotor in rotary direction and in pre-pressure direction influence the performances of the motor. The responses in the two directions have been investigated independently, because the two degrees of freedom are orthogonal in physics. To enhance drive force, rotation velocity and to reduce noise, the contact not only must be short enough but also must occur in the largest velocity area of the stator in the rotary direction of the rotor. The discussions presented in this paper can be used for guiding in the design of standing-wave-type ultrasonic motors.

Keywords Standing-wave · Intermittent contacts · Matching · Ultrasonic motors

1 Introduction

The ultrasonic motors featuring high holding torque and rapid response characteristics are promised to be used as a precise and accurate positioning actuator [1].They can be classified as traveling-wave-type and standing-wave-type according to their operating principles [2]. The standing-wave-type ultrasonic motors have advantage over travelling-wave-type ultrasonic motors in accurate positioning without feedback. However, the kinematical and dynamic matching between stator and rotor in a standing-wave-type ultrasonic motor is more important than in a travelling-wave-type ultrasonic motor, because the stator intermittently contact with the rotor, but continuous contact in a travelling-wave-type ultrasonic motor. It is very important when and where the rotor contacts with the stator, because the cross component of the velocity of the driving end of the stator continuously changes with time according to a rule of cosine. The slippage on the contacting interface is not expected, because it will make noise, reduce efficiency and lifetime. A good contact not only must be short enough but also should occur in the largest velocity area of the driving end on the stator in the rotation direction of the rotor. The contact locations depend on the frequency and the amplitude responses of the rotor in pre-pressure direction. Therefore, we will evaluate the contact durations by investigating the vibration of the rotor driven by the stator. Some conditions are required for enhance drive force, rotation velocity and to reduce noise. Finally, the maximal torque of load has been derived. The discussion is expected for guiding in the construction design of standing-wave-type ultrasonic motors.

J. Jin (✉) · C. Zhao
Precision Driving Laboratory,
Nanjing University of Aeronautics and Astronautics,
Nanjing 210016, People's Republic of China
e-mail: jjm345@sohu.com

2 Vibration of stator

An annular plate with three projections can be used for a stator of standing-wave-type ultrasonic motor as shown as the Fig. 1.

The Fig. 2 is a fragmented, diagrammatic elevational view of the bending vibration shape of B_{03} mode of the stator (subscripts 0 and 3 denote the number of nodal circles and the number of nodal diameters, respectively). *h* denotes

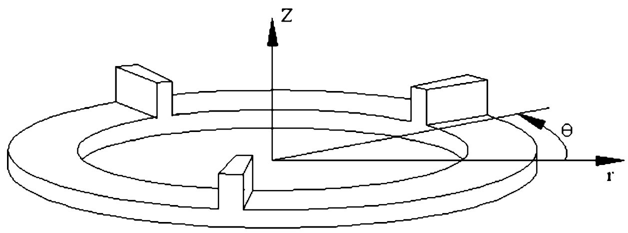


Fig. 1 Stator of standing-wave-type ultrasonic motor

the height of the projections and d denotes the thickness of the annular plate.

The resonance frequencies and displacements of disks or annular plates in free vibration have already been analysed [3]. The neutral plane displacement of the annular plate in z-direction can be written as

$$z_{\mathrm{mid}}(r,\theta,t)=\xi R(r)\sin(k\theta)\sin(\omega t+\alpha)$$

$$\delta=\arctan\frac{\partial z_{mid}}{r\partial\theta}$$

where, ξ is a coefficient related with driving voltages, $R(r)$ is Bessel function, k is the number of nodal diameter, ω is the resonant frequency and α is the response phase of the stator.

The displacement of the point P of the projection in θ -direction can be written as

$$\vartheta_P(r,\theta,t)=-\left(h+\frac{d}{2}\right)\frac{1}{r}\sin\delta$$

Since $\delta \ll 1$, $\sin\delta \approx \tan\delta$. We have

$$\vartheta_P(r,\theta,t)=-\left(h+\frac{d}{2}\right)\frac{\partial z_{\mathrm{mid}}}{r^2\partial\theta}$$
$$=-\frac{k}{r^2}\left(h+\frac{d}{2}\right)\xi R(r)\cos(k\theta)\sin(\omega t+\alpha)$$

and then the velocity of point P in θ -direction can be expressed as

$$\dot{\vartheta}_P(r,\theta,t)=\frac{\partial\vartheta_P(r,\theta,t)}{\partial t}$$
$$=-\frac{k}{r^2}\omega\left(h+\frac{d}{2}\right)\xi R(r)\cos(k\theta)\cos(\omega t+\alpha)$$

The displacement z_P and the velocity $\dot{z}_P$ of the point $\boldsymbol{P}$ of the projection in z-direction can be written as

$$z_P=z_{\mathrm{mid}}-\vartheta_P\tan\left(\frac{1}{2}\delta\right)\approx z_{\mathrm{mid}}$$

$$\dot{z}_P\approx\dot{z}_{\mathrm{mid}}=\omega\xi R(r)\sin(k\theta)\cos(\omega t+\alpha)$$

Because this work discusses the relationships between stator and rotor, assuming the response phase of the stator α equal to zero, the displacements z_P, ϑ_p and the velocities $\dot{z}_P, \dot{\vartheta}_p$ can be rewritten, respectively, as

$$z_P(r,\theta,t)=\xi R(r)\sin(k\theta)\sin(\omega t) \tag{1}$$

$$\dot{z}_P(r,\theta,t)=\omega\xi R(r)\sin(k\theta)\cos(\omega t) \tag{2}$$

$$\vartheta_P(r,\theta,t)=-\frac{k}{r^2}\left(h+\frac{d}{2}\right)\xi R(r)\cos(k\theta)\sin(\omega t) \tag{3}$$

$$\dot{\vartheta}_P(r,\theta,t)=-\frac{k}{r^2}\omega\left(h+\frac{d}{2}\right)\xi R(r)\cos(k\theta)\cos(\omega t) \tag{4}$$

3 Vibration and rotation of rotor

In general, a rotary standing-wave-type ultrasonic motor is mainly composed of piezoelectric elements, stator, rotor, and spring as shown in Fig. 3. In which, the pre-pressure F between the stator and the rotor is provided by the spring.

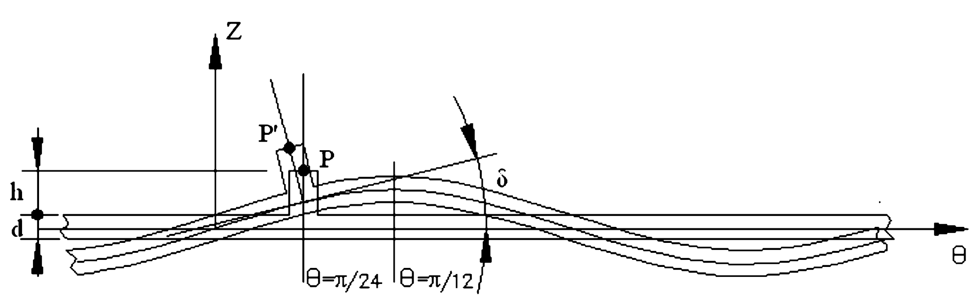
The Fig. 4 illustrated two freedoms of the rotor. One is rotation around the z-axis (θ_r), and the other is vibration along z-axis (z_r).

As mention above, the movements of the rotor in its rotation direction and in its pre-pressure direction can be independently investigated. The beginning time and the ending time of contact can be derived by discussing the response in z-direction.

To simplify problem, we have made the following assumptions,

a. The projections of the stator are rigid;
b. Vibration of the stator is not influenced by rotor [4–6], and defined,

Fig. 2 Fragmented, diagrammatic elevational view of the stator



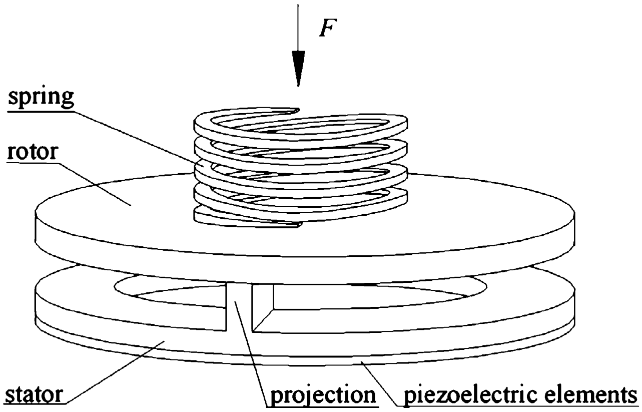


Fig. 3 Primary parts of a rotation standing-wave-type ultrasonic motor

c. The duration from beginning contact to next beginning contact is a operating period of motor;

d. The separate time is t₁ (or $t_1^{'''}, t_1^{''}, t_1^{'''}, \ldots$)in a period of operation $t_0 \sim t_0^{'}$(or $t_0^{'} \sim t_0^{''}, t_0^{''} \sim t_0^{'''}, \ldots$), namely, $t_0 < t_1 < t_0^{'}, t_0^{'} < t_1^{'} < t_0^{''}, t_0^{''} < t_1^{''} < t_0^{'''}, \ldots$.

The vibration of the rotor in z-direction can be described by a spring-mass system as shown in the Fig. 5.

In this figure, c_z and k_z denote the damping and the stiffness of the spring; c_{con} and k_{con} denote the contacting damping and the contacting stiffness of the rotor, respectively; m is the mass of rotor; $z_p(t)$ and $z_r(t)$ denote displacements of the projection and the rotor in z-direction, respectively.

The vibration of the rotor is described as two phases in an operating period. The first phase from t_0 (or $t_0^{'}, t_0, \ldots$) to t_1(or $t_1^{'}, t_1^{''}, \ldots$) is forced vibration by the projection of stator, the second phase from t_1 (or $t_1^{'}, t_1^{''}, \ldots$) to $t_0^{'}$ (or $t_0^{''}, t_0^{'''}, \ldots$) is free vibration under the initial displacement $z_r(t_1)$ (or $z_r(t_1)$, $z_r(t_1^{'}), \ldots$) and initial velocity $\dot{z}_r(t_1)$ (or $\dot{z}_r(t_1), \dot{z}_r(t_1^{'}),$).

According to Newton's second law, in the first operating period ($t_0 \sim t_0^{'}$), motion of the rotor is described as following, In the first phase $t_0<t<t_1$,

$$\begin{cases} m\ddot{z}_r + (c_z + c_{\mathrm{con}})\dot{z}_r + (k_z + k_{\mathrm{con}})z_r = f(t) \\ z_r(t_0) = z_0, \qquad \dot{z}_r(t_0) = v_0 \\ f(t) = c_{\mathrm{con}}\dot{z}_p + k_{\mathrm{con}}\left(z_p + \dfrac{k_{\mathrm{con}} + k_z}{k_{\mathrm{con}}} z_0\right) \end{cases} \tag{5}$$

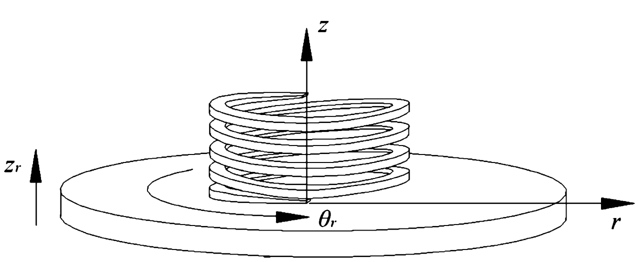


Fig. 4 Motions of the rotor

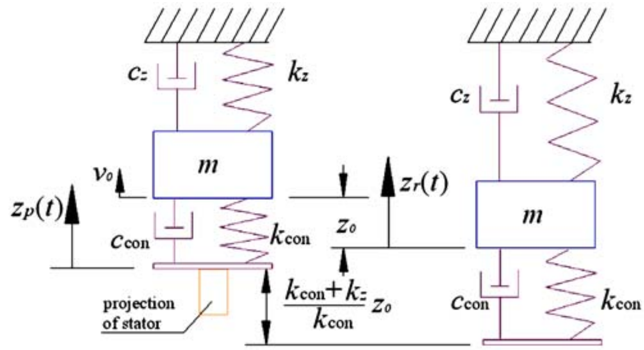


Fig. 5 Model of the vibration of rotor in z-direction

The solution of Eq. 5 is

$$z_r(t) = e^{-\zeta_{\mathrm{con}}\omega_{n,\mathrm{con}}(t-t_0)} \left[z_0 \cos\omega_{d,\mathrm{con}}(t-t_0) + \frac{v_0 + \zeta_{\mathrm{con}}\omega_{n,\mathrm{con}} z_0}{\omega_{d.\mathrm{con}}} \sin\omega_{d,\mathrm{con}}(t-t_0)\right] + \int_{t_0}^{t} h(t-\tau) f(\tau) d\tau \tag{6}$$

where $\zeta_{\mathrm{con}} = \frac{c_z + c_{\mathrm{con}}}{2\sqrt{m(k_z + k_{\mathrm{con}})}}$; $\omega_{n,\mathrm{con}} = \sqrt{\frac{k_z + k_{\mathrm{con}}}{m}}$; $\omega_{d,\mathrm{con}} = \sqrt{1 - \zeta_{\mathrm{con}}}\omega_{n,\mathrm{con}}$; $h(t-\tau) = \frac{1}{m\omega_{d,con}} e^{-\zeta_{con}\omega_{n,con}(t-\tau)} \sin\omega_{d,con}(t-\tau)$, $t \geq \tau$;

When the motor began start-up (t_0=0), the initial velocity v_0 equal to zero, and the initial displacement $z_0 = \frac{F}{k_z}$ rooted in the installation of motor for the pre-pressure force F.

Because separate time t_1 satisfies,

$$z_p(t) + \frac{k_{\mathrm{con}} + k_z}{k_{\mathrm{con}}} > z_r(t) \qquad t_0 < t < t_1$$

$$z_p(t) + \frac{k_{\mathrm{con}} + k_z}{k_{\mathrm{con}}} = z_r(t) \qquad t_1 = t$$

The separate time t_1 can be calculated by a step-by-step method, and $z_r(t_1)$, $\dot{z}_r(t_1)$ can be obtained at same time.

In the second phase $t_1 < t < t_0^{'}$, $f(t)$=0, the motion of the rotor is described as,

$$\begin{cases} m\ddot{z}_r + c_z\dot{z}_r + k_z z_r = 0 \\ z_r(t_1) = z_1, \dot{z}_r(t_1) = v_1 \end{cases} \tag{7}$$

Its solution is

$$z_r(t) = e^{-\zeta\omega_n(t-t_1)}\left[z_1 \cos\omega_d(t-t_1) + \frac{v_1 + \zeta\omega_n z_1}{\omega_d} \sin\omega_d(t-t_1)\right] \tag{8}$$

where $\zeta = \frac{c_z}{2\sqrt{mk_z}}$; $\omega_n = \sqrt{\frac{k_z}{m}}$; $\omega_d = \sqrt{1-\zeta}\omega_n$; $h(t-\tau) = \frac{1}{m\omega_d} e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau), t \geq \tau$.

The second encounter time $t_0^{'}$ can be determined by,

$$z_p(t) + \frac{k_{\mathrm{con}} + k_z}{k_{\mathrm{con}}} z_0 < z_r(t) \qquad t_1 < t < t_0^{'}$$

$$z_p(t) + \frac{k_{\mathrm{con}} + k_z}{k_{\mathrm{con}}} z_0 = z_r(t) \qquad t_0^{'} = t$$

and $z_r(t_0^{'})$, $\dot{z}_r(t_0^{'})$ can be obtained at same time.

Well then, $z_r(t_0'),\ \dot{z}_r(t_0')$ become the initial conditions of the motion rotor in the second operating period. By the way, the contact times $(t_0, t_0', t_0'', t_0''', \cdots)$ and the separate times $(t_1, t_1', t_1'', t_1''', \cdots)$ can be calculated by step-by-step method.

For rotation around the z-axis, Newton's second law applied to the rotor gives [7]

$$\begin{cases} J\dfrac{d\dot{\vartheta}_r}{\mathrm{d}t} + c_\theta\dot{\vartheta}_r = T_a(t) - T_l \\ \qquad \dot{\vartheta}_r(t_0) = v_{r0} \end{cases} \tag{9}$$

where $T_a(t)$ is the torque from the stator; T_l is the torque from load; J is the sum of inertia moment of the rotor and load; c_θ is the coefficient of damping.

$$\dot{\vartheta}_r(t) = v_{r0}e^{-\frac{c_\theta}{J}(t-t_0)} + e^{-\frac{c_\theta}{J}(t-t_0)}\int_{t_0}^{t}\frac{T_a(t)-T_l}{J}e^{\frac{c_\theta}{J}(t-t_0)}\mathrm{d}t \tag{10}$$

In the contact duration, a obstruction torque is possible if the velocity of point $\boldsymbol{P}$ in θ -direction and the rotate speed of the rotor did not meet the relationship,

$$\dot{\theta}_P(t) \geq \dot{\theta}_r(t), \quad t_0 \leq t \leq t_1, \quad t_0' \leq t \leq t_1', \quad t_0'' \leq t \leq t_1'', \ldots \tag{11}$$

4 Matching between stator and rotor

According to the equation (1–4), the relationship between the velocities and the displacements can be expressed as Fig. 6. It is obvious that $\dot{z}_P$ always achieves its maximum in the vibrating equilibrium location z_0, namely $z_p(t)$=0, as shown in Fig. 6(a), and $\dot{\theta}_P$ achieves its maximum in the vibrating equilibrium location z_0 too, as shown in Fig. 6(b). So, we expect that the stator always contact the rotor near z_0.

The perfect loci of the stator and the rotor are illustrated in Fig. 7.

In this case, the contact will take place in the largest velocity zone of the point $\boldsymbol{P}$ in the rotation direction. In the same time, the condition described of the equation (11) will be met also, since

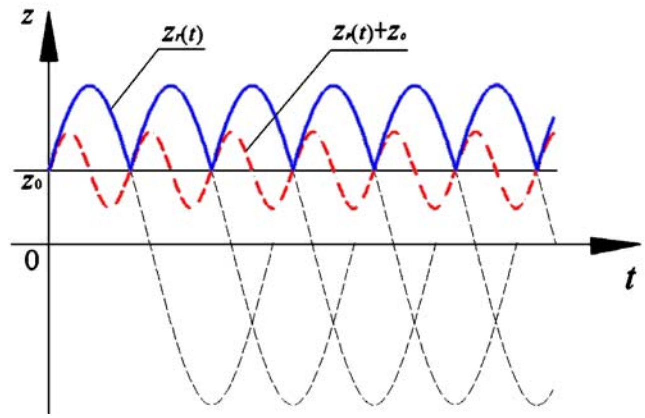


Fig. 7 Loci of the rotor and the stator in z-direction

$$\dot{\theta}_P\left(t_0'\right) = \dot{\theta}_P(t_0) \geqslant \dot{\theta}_P(t_1) = \dot{\theta}_r(t_1)$$
$$\dot{\theta}_P\left(t_0''\right) = \dot{\theta}_P\left(t_0'\right) \geqslant \dot{\theta}_P\left(t_1'\right) = \dot{\theta}_r\left(t_1'\right)$$
$$\cdots$$

By adjusting the stiffness k_z or mass m, the following equation can be met.

$$z_0 = z_r\left(t_0'\right)$$

If the vibration cycle $\left(T = \frac{2\pi}{\omega}\right)$ of the stator equal to the time $(t_0' - t_0)$, the second impact between the rotor and the stator will occur in the balance location (z_0) of stator. Thus, new initial conditions are initial displacement z_0 and initial velocity $v_0' = \dot{z}_r(t_0')$.

In fact, if $z_0 = z_r(t_0')$, the consequent contact times $t_0'', t_0''', \cdots$ will not satisfy the following equations.

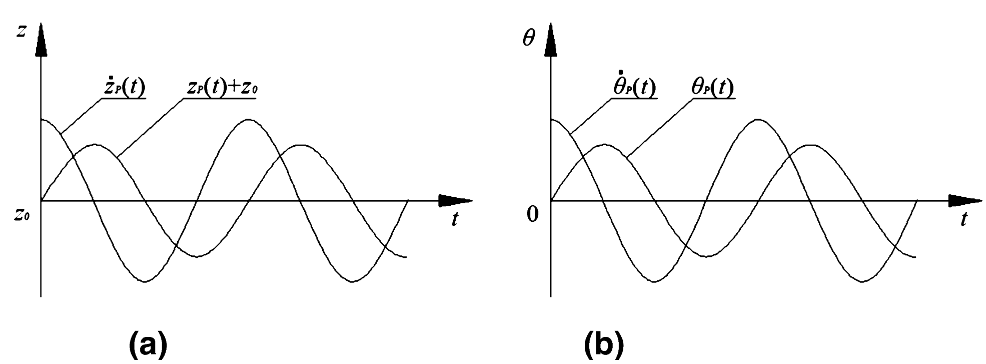
$$z_0 = z_r\left(t_0''\right),\ z_0 = z_r\left(t_0'''\right), \cdots$$

and separate times $t_1'', t_1''', \cdots$ will change also.

Two cases could take place in the third contact. One is $z_0 + \Delta z'' = z_r(t_0'')$ as shown in the Fig. 8(a), and the other is $z_0 - \Delta z'' = z_r(t_0'')$ as shown in the Fig. 8(b).

Both cases will result in decrease of the velocities $\left(\dot{z}_P, \dot{\vartheta}_P\right)$ at t_0''. The span of contact will increase with $\Delta z''$

Fig. 6 (**a**) Displacement z_P and velocity $\dot{z}_P$ of the top point P of the projection in Z-direction. (**b**) Displacement θ_P and velocity $\dot{\theta}_P$ of the top point P of the projection in θ-direction



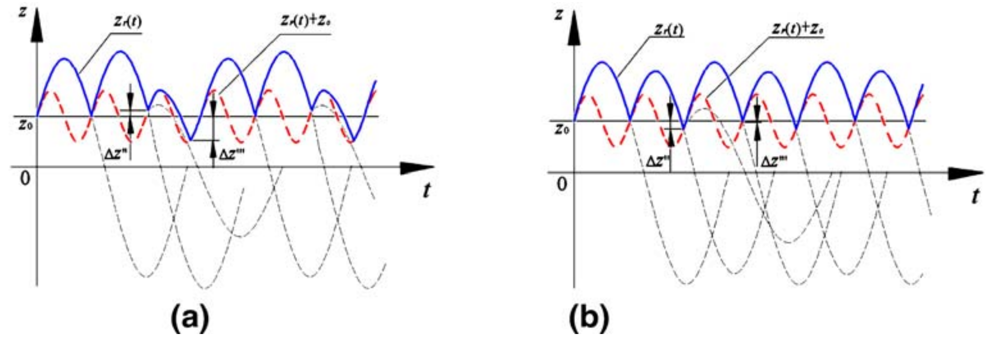


Fig. 8 The motions in z-direction in consideration of Δz_0

and the condition equation (11) described may not be met. Therefore, the deviations (Δz, $\Delta z'$, $\Delta z''$, $\Delta z'''$,...) are key parameters for the performance of the motor. It is necessary to maintain the deviations (Δz, $\Delta z'$, $\Delta z''$, $\Delta z'''$, ...) small enough by optimizing the parameters m, k_z and z_0.

5 Torque and rotary speed of motor

In fact, although stick-slip always exists in the interface between the stator and the rotor, the status without slippage has been an important aim in ultrasonic motors. We can estimate the rotor's angular speed, at the contact radius of the representing the tangential velocity of the contact points of the stator [8]. The assumption that there are not slippage during the contact of the stator and the rotor can give an upper limit of torque or rotary speed. On the assumption, For the first operating period $\left(t_0 \sim t_0^{'}\right)$, the torque can be expressed as

$$T_a(t) - T_l = J\frac{d}{dt}\dot{\vartheta}_p + c_\theta\dot{\vartheta}_p, \qquad t_0 < t < t_1 \tag{12}$$

Where $\dot{\vartheta}_P(t) = -\frac{k}{r^2}\omega\left(h+\frac{d}{2}\right)B\cos(\omega t), B = \xi R(r)\cos(k\theta)$

Substitute into Eq. 10, motion of the rotor is described as,

The first phase $t_0<t<t_1$,

$$\dot{\vartheta}_r(t) = e^{-\frac{c_\theta}{J}(t-t_0)}\left\{v_{r0} - \dot{\vartheta}_p(t_0)\right\} + \dot{\vartheta}_p(t) \tag{13}$$

The second phase $t_1 < t < t_0^{'}$,

$$\dot{\vartheta}_r(t) = e^{-\frac{c_\theta}{J}(t-t_0)}\left\{v_{r0} - \dot{\vartheta}_p(t_0)\right\} + e^{-\frac{c_\theta}{J}(t-t_1)}\left\{\dot{\vartheta}_p(t_1) + \frac{T_l}{c_\theta}\right\} - \frac{T_l}{c_\theta} \tag{14}$$

So, the rotary velocity $\dot{\vartheta}_r(t)$ can be calculated by step-by-step method.

The torque T_l of load attenuated rotation velocity $\dot{\vartheta}_r(t)$ after the stator left the rotor. To insure that the rotor did not reverse before the coming contact, the maximal torque $T_{l,\max}$ of load is limited,

$$T_{l,\max} = \frac{c_\theta}{1 - e^{-\frac{c_\theta}{J}\left(t_o^{'}-t_1\right)}} e^{-\frac{c_\theta}{J}\left(t_o^{'}-t_0\right)}\left\{v_{r0} - \dot{\vartheta}_p(t_0)\right\} + e^{-\frac{c_\theta}{J}\left(t_o^{'}-t_1\right)}\dot{\vartheta}_p(t_1) \tag{15}$$

Correspondingly, the average rotation velocity $\overline{\dot{\vartheta}_r(t)}$ from t_0 to $t_0^{'}$ can be written as

$$\overline{\dot{\vartheta}_r(t)} = \frac{1}{t_0^{'} - t_0}\int_{t_0}^{t_0^{'}} \dot{\vartheta}_r(t)dt = \frac{1}{T}\int_{t_0}^{t_0^{'}} \dot{\vartheta}_r(t)dt = \frac{\omega}{2\pi}\int_{t_0}^{t_0^{'}} \dot{\vartheta}_r(t)dt \tag{16}$$

The maximal statically frictional force $f_{\theta,\max}$ on contacting surface can be expressed

$$f_{\theta,\max} = \mu F = \mu k_z(z_0 - \Delta z_{\max})$$

Where μ denote the statically frictional coefficient; $\Delta z_{\max}$ denote the largest deviation.

To avoid slipping of the contact surface, the following condition must be satisfied

$$T_{l,\max} \le r\mu k_z(z_0 - \Delta z)$$
$$\mu = \frac{f_{\theta,\max}}{k_z(z_0 - \Delta z)} \geqslant \frac{T_{l,\max}}{rk_z(z_0 - \Delta z_{\max})} \tag{17}$$

6 Conclusion

From the preliminary discussion, the begin time and the end time of contact can be derived by investigating the vibrating of rotor in z-direction. In the contact duration, a obstruction torque is possible if the velocity of point ***P*** in θ-direction of stator and the rotate speed of the rotor did not meet the equation (11). To enhance drive force, rotation velocity and to reduce noise, the contact not only must be short enough but also must occur in the largest velocity area (near z_0) of the top of the projections on the stator in the rotary direction of the rotor. It is necessary to maintain deviations ($\Delta z, \Delta z', \Delta z'', \Delta z'''$, ...) small enough by optimizing the parameters m, k_z or z_0. The maximal torque $T_{l,\max}$ of load and the average rotation velocity $\overline{\dot{\vartheta}_r(t)}$ can be calculated. To avoid slipping of the contacting interface, the equation (17) must be satisfied. These results can be used for guiding in the design of standing-wave-type ultrasonic motors.

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